

Research on Maximizing Game Benefits with Limited Resources Based on Optimal Control Theory

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Abstract: According to the rules of the game, the linear programming model, multi-objective optimization model and intelligent algorithm in operational research are used to solve the problems of resource carrying and maximum profit. The first and second levels have the same data except different maps, so they are abstracted to solve general problems. When the weather is known, to seek the maximum benefit, first simplify the map, build the shortest path model, and find the shortest path from the starting point to the mine and village with the help of the toolbox in MATLAB; According to the known conditions, the single objective function and related linear constraints are determined and solved by simplex method; Finally, we get the following conclusion: if players want to achieve the maximum profit under the first and second level, they must adopt the strategy of starting point to mine and supplying in the village, at the same time, the mining days reach 8 days in the first level and 11 days in the second level, at this time, the optimal profits of the first and second level are 11880 and 11275 yuan. According to weather information, assuming that the distribution of weather obeys multi-item distribution, Bayesian method and maximum likelihood estimation in mathematical statistics are used to determine the parameters of multi-item distribution, pseudo-random is used to simulate the distribution of weather, and finally Monte Carlo algorithm is used to choose strategies and test the feasibility of the model, it is concluded that the third level strategy is to go to the end point directly without mining as the optimal solution, and the fourth level strategy is to start from the mine and supply it twice before leaving, and supply 246 boxes of food and 235 boxes of water for the first time.

1. Introduction

This problem is mainly launched under the background of the game of crossing the desert, players buy food and water with their limited funds with a map, and there is a fixed supply station in the middle, so as to supplement funds and resources, reach the end point within the specified time and get the maximum benefit.

According to the rules of the game, the following questions are obtained:

- 1) When there is only one player and all the weather is known, how to optimize the carrying of resources and arrange the itinerary in the first two passes, so as to maximize the benefits?
- 2) When there is only one player and only the weather of the day is known, how can we pass the third and fourth levels to win?

2. Establishment of linear programming model

In order to maximize our remaining funds, we should either clear customs as quickly as possible, or reach the mine as quickly as possible, [1] and reach the destination from the mine as quickly as possible, so as to make our income as large as possible. According to this idea, we simplify the map and get the path graph as shown in Figure 1.

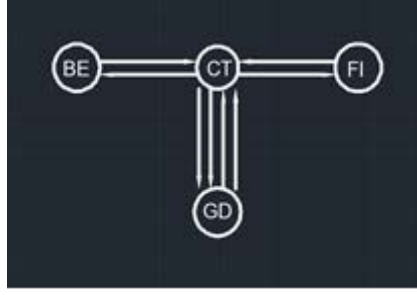


Figure 1. Map Path

According to the first level, discuss the first situation, and clear customs as quickly as possible, according to the rules of the game, customs clearance can take up to three days without considering the weather, according to the weather conditions of 30 days, the time to reach the destination is still three days, so on the 0 th day, we can adopt the following strategies:

Buy water and food:

$$S = \begin{pmatrix} sw \\ sf \end{pmatrix} = \begin{pmatrix} 42 \\ 38 \end{pmatrix},$$

The cost of water and food in three days:

$$S^T P = (42 \quad 38) \begin{pmatrix} 5 \\ 10 \end{pmatrix} = 590yuan < 10000yuan$$

Three days of water and food load:

$$S^T P = (42 \quad 38) \begin{pmatrix} 3 \\ 2 \end{pmatrix} = 202kg < 1200kg$$

Therefore, when the player finally reaches the finish line, his remaining funds are 9410 yuan.

According to the first level, in the second case, according to the path in fig.1 and the rules of the game, the components of the player's remaining funds are the remaining funds after purchasing food and water and the mine income funds, so the objective function is obtained:

$$\max M = m_0 - S_{BE}^T P - 2S_{CT}^T P + \frac{P^T}{2} (S_{BE} + S_{CT} - C(2A + 3B + R))$$

In which:

$$= 1 : 30, .e_j = \begin{cases} 0, & \text{otherwise;} \\ 1, & \text{rest} \end{cases};$$

$$S_{BE} = \begin{pmatrix} sw \\ sf \end{pmatrix};$$

$$A = (a_1 \quad a_2 \quad \cdots \quad a_{30})^T; a_i = \begin{cases} 1, & \text{walk} \\ 0, & \text{otherwise;} \end{cases};$$

$$B = (b_1 \quad b_2 \quad \cdots \quad b_{30}); b_j = \begin{cases} 1, & \text{mining} \\ 0, & \text{otherwise} \end{cases}$$

The following is to determine the constraints. Before determining the constraints, it is necessary to analyze the path of BE to CT and the time required to determine whether the player's supply point chooses the starting point or the village. Therefore, we choose to solve the shortest distance and path of the designated point with the help of Graphshortestpath function in MATLAB. [2]

Combined with the weather, we can judge that if players do not buy 1200kg of food and water at the starting point, they may need to return to the starting point midway, thus greatly reducing the mining time, from the map, it takes at least two days for the shortest path from the village to the mine, while it takes 10 days from the starting point to the mine, therefore, players should not choose the starting point as the replenishment point in order to achieve the best profit, so players should buy food and water up to the upper limit at the starting point, so the constraint conditions are obtained.

$$S_{BE}^T \begin{pmatrix} 3 \\ 2 \end{pmatrix} = (sw \quad sf) \begin{pmatrix} 3 \\ 2 \end{pmatrix} = 3sw + 2sf = L_0$$

While passing through the village in the middle, the player continues to fill up the supply, when arriving at the mine, the player continues to mine, at this time, the player has two choices: one is to end the game when the remaining amount of food and water is enough to return to the finish line, the

other is to supply after mining, and then come back to continue mining, and finally go back to the village to supply.

Obviously, according to the map, no matter how you go from the mine site, regardless of the weather, it takes at least 5 days for the player to get from the mine site to the end point. From the previous analysis, we can know that from the starting point to the mine point, from the mine point to the end point. It takes at least 15 days in total, and a single replenishment requires at least 4 days. If replenishment is 3 times or more, the mining time is only no more than 3 days. In order to increase the mining time as much as possible, we can get from the above analysis, replenishment times $l > 0$, but $l < 3$.

According to the replenishment relationship, we can get the constraints on replenishment:

$$\sum_{i=1}^l S_{CT_i} = S_{CT}$$

In order to ensure that there is enough water and food every day during the game, the constraints are as follows:

$$\begin{aligned} S_{BE} - CQ_i(2A + 3B + R) &\geq 0, \forall 1 \leq i < n_1; \\ S_{BE} + S_{CT_1} - CQ_i(2A + 3B + R) &\geq 0, \forall n_1 \leq i < n_2; \\ S_{BE} + S_{CT_1} + \dots + S_{CT_l} - CQ_i(2A + 3B + R) &\geq 0, \forall n_l \leq i < n_{l+1}; \end{aligned}$$

According to the game deadline, get the constraint conditions:

$$\|A + B + R\| \leq 30$$

To sum up, the following constraints are obtained:

$$\left\{ \begin{array}{l} \max M = m_0 - S_{BE}^T P - 2S_{CT}^T P + \frac{P^T}{2} (S_{BE} + S_{CT} - C(2A + 3B + R)) \\ s. t. S_{BE}^T \binom{3}{2} = (sw \quad sf) \binom{3}{2} = 3sw + 2sf = L_0 \\ \sum_{i=1}^l S_{CT_i} = S_{CT} \\ S_{BE} - CQ_i(2A + 3B + R) \geq 0, \forall 1 \leq i < n_1; \\ S_{BE} + S_{CT_1} - CQ_i(2A + 3B + R) \geq 0, \forall n_1 \leq i < n_2; \\ S_{BE} + S_{CT_1} + \dots + S_{CT_l} - CQ_i(2A + 3B + R) \geq 0, \forall n_l \leq i < n_{l+1}; \\ \|A + B + R\| \leq 30 \end{array} \right.$$

The simplex method is used to solve the problem, and the results are as follows:

In order to achieve the maximum profit, players must adopt the strategy of starting point to mine and supplying in the village, at the same time, the mining days reach 8 days in the first level and 11 days in the second level, at this time, the optimal profits of the first level are 11880 and 11275 yuan.

3. Monte Carlo algorithm

Monte Carlo sampling method is a numerical simulation method with probability phenomenon as the research object, which is suitable for computer simulation test of discrete systems. Therefore, it is easy to detect the reliability of the model for computer simulation. [3]

Due to the uncertainty of weather conditions, we use Bayesian statistics to estimate the weather, namely:

$$\begin{aligned} \theta_1 = P(k_i = 1) &= \frac{N(\text{sunny})}{N(\text{sunny}) + N(\text{megathermal}) + N(\text{sand})}, \\ \theta_2 = P(k_i = 2) &= \frac{N(\text{megathermal})}{N(\text{sunny}) + N(\text{megathermal}) + N(\text{sand})}, \end{aligned}$$

$$\theta_3 = P(k_i = 3) = \frac{N(sand)}{N(sunny) + N(megathermal) + N(sand)}$$

According to the third condition, there is no sandstorm, which is estimated by maximum likelihood

$$\widehat{\theta}_1 = \frac{3}{8}, \widehat{\theta}_2 = \frac{5}{8}$$

According to the fourth condition, there is no sandstorm, which is estimated by maximum likelihood

$$\widehat{\theta}_1 = \frac{3}{8} \times 0.9, \widehat{\theta}_2 = \frac{5}{8} \times 0.9, \widehat{\theta}_3 = 0.1$$

When the weather is sunny, the minimum cost of moving is 110, when the weather is hot, if you choose to rest, the cost of moving is 245 when the next day is sunny, and the cost of moving at high temperature is 270, which is not much different. When mining, choose to mine, even if the weather is the worst sandstorm, you can still make a profit, that is, the choice has nothing to do with the weather. In the programming, we need to judge the mining time, at this time, we need the weather data, and we still use the third level Monte Carlo algorithm to test the feasibility and optimal conditions of this strategy. According to the general idea, the shortest path model is used to optimize the map, and the following conclusions can be obtained

1) From start point (BE) to mine (GD), mining to village (CT) to mine (DG), mining to village (CT) to mine (DG) to end point (FI)

2) From start point (BE) to village (CT) to mine (DG), mining to village (CT) to mine (DG), mining to end point (FI).

According to the prediction of mining time, it can take about 5 days to dig from the starting point (BE) to the mine (GD), but 9 days after replenishment, 5 days from the starting point (BE) to the mine (GD) or to the village (CT), and 3 days to leave, if the second option is chosen, the materials after sufficient replenishment will not be used up in mining the next day, namely these two kinds.

Using Monte Carlo method, in order to ensure 95% possibility of customs clearance, it is calculated that it is necessary to make the load reach 720kg.

(1) The inspection of third level

Step 1: Simulate the last 10 days according to the weather conditions of the previous days for 20,000 times.

Step 2: Record the remaining funds of the two schemes under different weather conditions.

Step 3: Draw a map according to the remaining funds.

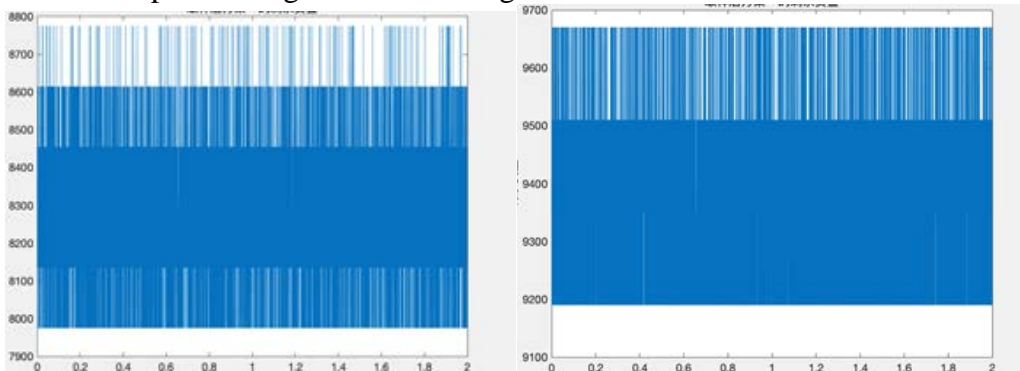


Figure 2. Remaining funds

By drawing, it is found that the number of scheme 1 is less than 9,200, and the number of scheme 2 over 8,800 is not better than scheme 1, which also meets our model.

(2) The inspection of fourth level

We modeled the Montacalo algorithm in the third level to calculate 20,000 sets of weather, and according to these weather conditions,

Here, we set the starting point when the player has less than one day of sandstorm and three days of high temperature.

$$P(k_i \neq 3, k_{i+1} \neq 3) = P(k_i \neq 3)P(k_{i+1} \neq 3) = 0.81$$

$$P(k_i = 3, k_{i+1} \neq 3) = 0.9 \times 0.1 = 0.09$$

$$P(\text{At most one sandstorm in two days}) = 0.99$$

When one day sandstorm and two days high temperature consume, players must return to the village to replenish materials. Whether or not to return to the village depends on the number of days left. In the simulation program, purple represents mine, yellow represents desert, blue represents village and green represents end point.

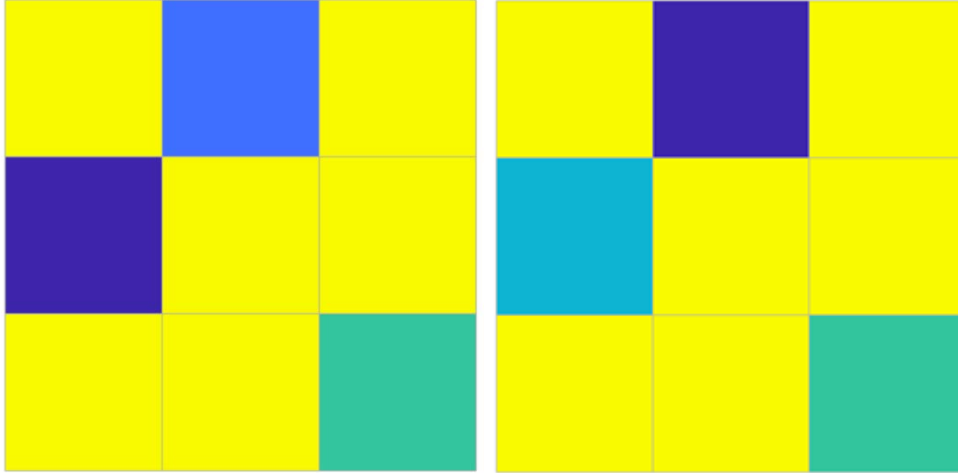


Figure 3. Players in the mine and village

When players return to the village for the first time, they must be filled with supplies

After many simulations, it is found that the second continuous mining can be carried out for 7-9 days when the village supplies are fully loaded for the first time, so as to achieve the highest profit. Let the cell (player) return to the mine and return to the situation just now, and judge the choice according to the remaining time.

It is found that 95% of the samples are larger than 480kg, which is in the most stable income, in the last time, it is not required to supply full, and only 720kg of materials can be supplied.

4. Conclusion

In the case of a player, the weather is all known, according to the map of the first level, we must consider how to achieve the best resources if we want to get the maximum benefit from customs clearance. In order to realize the maximum profit, the basic profit can make up for the consumption of mining. We can supplement the funds through the mine, in order to make the mining time longest, we try to reduce the time from the starting point to the mine, therefore, we use the Graphshortestpath function in MATLAB to calculate the shortest path, and then build a planning model to solve it. For the second level, we adopt the same thinking and solve it. The only difference is that different maps make different choices. Based on this, we discuss the different schemes in order to compare them and determine the optimal solution.

In the third and fourth level, the maximum likelihood estimation is used to calculate the possibility of weather occurrence, and pseudo-random weather generation is used to solve the weather problem. For the third level, according to the map, after simple estimation, there are two choices. One is that the player chooses to go to the mine and then leave for the end point, the other is that after this simplification, the path problem mainly comes down to the problem from the starting point to the mine point and then to the end point. The shortest path here is solved by the same algorithm as the first question; For the fourth level, players need to consider whether to go to the mine or the village first, so it is still reduced to two path optimization problems after simplification; Finally, the computer simulation is used to set the corresponding parameters and solve the final result.

Linear programming model can be applied to resource allocation and production decision, while Monte Carlo algorithm can be extended to medical diseases, military decision and other fields.

References

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